

Consider the Banach space $\ell^\infty(\mathbb{N})$ with the supremum norm $\| \cdot \|_\infty$. Define the linear maps

$$L_n : \ell^\infty(\mathbb{N}) \rightarrow \mathbb{C} : L_n(f) = \frac{1}{n} \sum_{k=n+1}^{2n} f(k) .$$

1. Prove that (L_n) is a sequence in the unit ball of $\ell^\infty(\mathbb{N})^*$.
2. Let $L \in \ell^\infty(\mathbb{N})^*$ be a weak* limit point of the sequence (L_n) . Why does such an L exist?
3. Prove that L is a Banach limit in the sense of Theorem 3.7.