Take home exam Functional Analysis

Project 2. Schur's theorem on weakly convergent sequences in $\ell^1(\mathbb{N})$.

- Read carefully the instructions on Toledo, including the due date for the solutions.
- If something is unclear, please feel free to ask me, during the lecture or by e-mailing me at stefaan.vaes@wis.kuleuven.be.
- If you cannot prove or solve (part of) a question, continue to the next question and use the non-proven statement as a black box if needed.

1. Prove that on a Banach space X the weak topology coincides with the norm topology if and only if X is finite dimensional. (See exercise 3 in Lecture 7 for a hint.)
2. Let X be an infinite dimensional Banach space. Prove that there exists a *net* (x_i) in X such that (x_i) converges to 0 in the weak topology, but does not converge to 0 in the norm topology.
3. Let $p > 1$. Give an example of a *sequence* in $\ell^p(\mathbb{N})$ that converges to 0 in the weak topology, but not in the norm topology. Prove your answer.
4. Write in your own words a complete proof of the following theorem of Schur: if a *sequence* in $\ell^1(\mathbb{N})$ converges to 0 in the weak topology, then it converges to 0 in the norm topology. You can read a proof of this theorem in Conway's book, Chapter V, Proposition 5.2. In your final text you may of course use theorems from the lecture notes, but you have to prove everything else.