Examen
Statistische Mechanica bij Evenwicht
14 November 2013, 10u30

The score is calculated to 20 points!

2.5 points

Diffusion in 1d

Consider N diffusing particles in one dimension and let D be the diffusion coefficient. Let us suppose that at time $t = 0$ the concentration is

$$c(x,0) = \frac{N}{a\sqrt{\pi}}\, e^{-\dfrac{x^2}{2a^2}} \tag{1}$$

where a is given. Calculate τ the time is takes for the concentration in $x = 0$ to reach half of the initial value:

$$c(0,\tau) = \frac{N}{2a\sqrt{\pi}} \tag{2}$$

2.5 points

Chemical Potential of ideal gas

Compute $\mu(N,V,T)$ the chemical potential of an ideal gas using the canonical ensemble[1] and the grand canonical ensemble and show that the two quantities match.

2.5 points

Gas in gravitational field

Consider an ideal gas of N particles in a gravitational field with potential $V(z) = mgz$. How does the density depend on the height z?

Consider two heights $z_1 \gg z_2$. Which of the following statements is true (justify!):

(a) The average velocity of particles located around z_1 is much higher than that of particles in z_2.

(b) The average velocity of particles located around z_2 is much higher than that of particles in z_1.

(c) Particles will have the same average velocity independent on the height.

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[1] Recall that

$$\mu = \left.\frac{\partial F}{\partial N}\right|_{V,T}$$

and use the Stirling approximation $\log N! \approx N \log N - N$

2.5 points

Energy Fluctuations

Show that:

$$\left.\frac{\partial^2 \log Z}{\partial \beta^2}\right|_{N,V} = \langle E^2 \rangle - \langle E \rangle^2$$

where $Z(N, V, T)$ is the canonical partition function. As a consequence show that the specific heat at constant volume is given by

$$c_V = \frac{\langle E^2 \rangle - \langle E \rangle^2}{k_B T^2}$$

5 points

Relativistic gas

Consider a system of N relativistic particles in a volume V and at a temperature T. In the limit of small masses the Hamiltonian is given by:

$$\mathcal{H} = c|\vec{p}|$$

a) Compute the canonical partition function for this system and derive the energy and specific heat. Show that the result is consistent with the equipartition theorem.

b) Determine the pressure as a function of volume, temperature and number of particles.

5 points

Second Virial Coefficient of Argon

Figure 1 shows experimental data for the second virial coefficient for Argon plotted as a function of the inverse temperature $1/T$. In the high temperature region, the experimental data turn out to be well-fitted by a parabola:

$$b_2(T) = A - \frac{B}{T} - \frac{C}{T^2} \tag{3}$$

Suppose that the system could be described by a square-well interparticle potential

$$\phi(r) \quad = \quad \begin{cases} \infty & 0 < r < \sigma \\ -\varepsilon & \sigma < r < \sigma' \\ 0 & \quad r > \sigma' \end{cases} \tag{4}$$

which is a function of the parameters ε, σ and σ'.

a) Express the three parameters ε, σ and σ' as a function of A, B and C.

b) Find the Boyle temperature as a function of A, B and C.

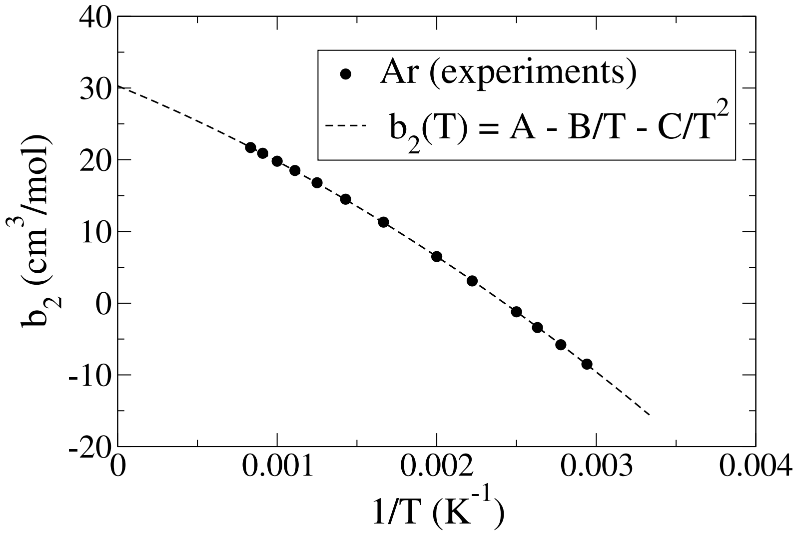

Figure 1: Circles: Second virial coefficient for Argon plotted as a function of the inverse temperature (source R.B. Stewart and R.J. Jacobsen *J. Phys. Chem. Ref. Data* **18**, 639 (1989).). Dashed line: parabolic fit of the data.