

QFT Exam

7 Januari 2019

1 Question 1 : Classical Fields

Given a Dirac spinor field ψ and two real boson fields ϕ_1 and ϕ_2 , we have a Lagrangian density

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 + \frac{1}{2} \partial_\mu \phi_2 \partial^\mu \phi_2 - \frac{m^2}{2} \phi_1^2 - \frac{\tilde{m}^2}{2} \phi_2^2 - \tilde{m}^2 \phi_1 \phi_2 + i \bar{\psi} \not{\partial} \psi - M \bar{\psi} \psi + i \lambda \phi_1 \bar{\psi} \gamma_5 \psi$$

with $\lambda, M, m, \tilde{m} \in \mathbb{R}$ different from zero and $m^2 > \tilde{m}^2$.

1. What is the dimension of λ ? Why is there a factor i in the last interaction term?
2. How should ϕ_1 and ϕ_2 transform under Lorentz transformations so that \mathcal{L} is invariant under the full Lorentz group.
3. What are the asymptotic states of this theory?

2 Question 2 : Gauge Invariance of Feynman amplitudes

1. Why is $e^+e^- \rightarrow \gamma$ not a physical process?
2. Look at the physical process $e^+e^- \rightarrow \gamma\gamma$. The positron has momentum and helicity (p_1, r_1) and the electron has (p_2, r_2) . The photons have (k_1, s_1) and (k_2, s_2) . Give the two Feynman diagrams that describe this process in leading order. Write down the Feynman amplitudes explicitly.
3. Replace in the preceding expressions $\epsilon_{s_1}(k_1)$ with k_1 and show that the two terms cancel each other.
4. Explain why this happens.