QFT Exam

Tuesday 8 January 2019

Question 1: Vectorfields

Consider a vectorfield A^μ with a Lagrangian density:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu$$

where

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

and $m \in \mathbb{R}$ larger than zero.

1. Show that this Lagrangian density is not gauge invariant.
2. Find the equation of motion by varying the action. Show that even though there is no gauge invariance, the Lorentz condition $\partial_\mu A^\mu$ is implied.
3. Solve for the general equations of motion and explain the results. Compare with the massless case.
4. Find the Green's function of the equation of motion (in momentum-space) and use this to determine the propagator. Examine what happens to propagator in the UV limit and compare the effect to what was discussed during the lectures. (Hint from after the exam: It is not the aim to calculate the Green function using the Feynman propagator, instead think of it as taking the Fourier transformation of the inverse of the equation of motion. [For an explanation read Peskin & Schroeder on the Klein-Gordon and Dirac Propagator.] In the UV limit consider the effect on radiative corrections.)

Question 2:

Consider Compton scattering of positrons $e^+\gamma \to e^+\gamma$ in leading order of the Feynman diagrams. The incoming positron has (momentum, helicity) (p_1,r_1), the incoming photon has (momentum, polarisation) (k_1,s_1), the outgoing positron has (momentum, helicity) (p_2,r_2) and the outgoing photon has (momentum, polarisation) (k_2,s_2) finally.

1. Give the two Feynman diagrams that describe this process in leading order. Write down the Feynman amplitudes explicitly, carefully labelling each operator completely.
2. Replace in the preceding expressions $\varepsilon_{s_2}(k_2)$ with k_2 and show that the two contributions cancel each other.
3. Explain.