Examen Statistische mechanica 30 januari 2020

1 Oral

1.1 classical

Virial expansion
For a gas we define $n = \frac{N}{V}$
The virial expansion is defined as follows.

$$P = nk_bT(1 + b_2n + ...)$$

Also define a potential:

$$\phi = \begin{cases} +\infty & r < \sigma \\ \epsilon & \sigma < r < 2\sigma \\ 0 & r > 2\sigma \end{cases}$$

Discus the temperature dependance of b_2 and use explicit calculations with $\phi(r)$.
Discus the case $\epsilon > 0$, $\epsilon < 0$, $\epsilon = 0$
Mondelinge bijvraag: Wat is de dimensie van b_2

1.2 Quantum

De blackbody die op sommige andere examens staat

2 Written

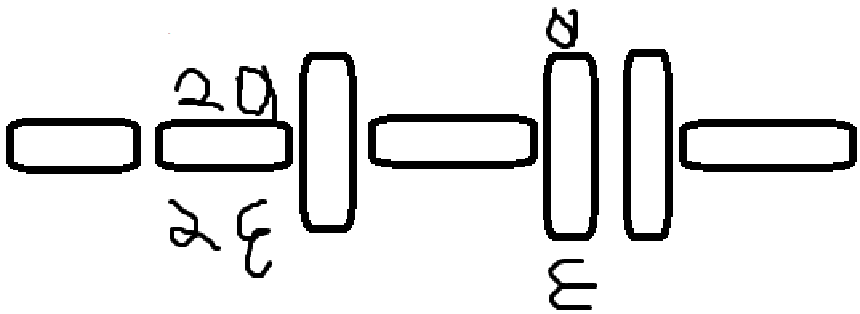
2.1 Classical

Speed of particles of LJ fluid (5pts)

$$\phi_{\mathrm{lJ}}(\mathrm{r}) = \epsilon\left[\left(\frac{\sigma}{\mathrm{r}}\right)^{12} - \left(\frac{\sigma}{\mathrm{r}}\right)^{6}\right]$$

Vind v^*, the most likely value of the speed of the particles speed.

Polymer model(7pts)
Consider a rigid polymer consisting of N monomers. Each monomer can be found in 2 states with energies ϵ and 2ϵ and length a and 2a as shown in the figure. Calculate average length <L> and variance σ_{L}^2 = <L^2> - <L>^{2}



Classical paramagnetism(8pts)
Assume each particle carries magnetic momentum $\vec{\mu}$, wich is fixed of magnitude, but can assume a random orientation in 3D. The hamiltionan for N particles is

$$\mathcal{H} = -\sum_{i=1}^{\mathrm{N}} \vec{\mu}_{\mathrm{i}} \cdot \vec{\mathrm{H}}$$

Where $\vec{\mathrm{H}}$ is the magnetic field and $\vec{\mu}_{\mathrm{i}}$ the magnetic moment of particle i

a) Calculate configurational partition function by integrating over all possible orientations of $d\vec{\mu}$, while keeping their orientations fixed $\|\vec{\mu}\| = \mu$(hint: use a coordinate system where H is in the z direction)

b) calculate total average magnetic moment

$$\vec{\mathrm{M}} = \sum_{\mathrm{i}\,=\,1}^{\mathrm{N}} < \vec{\mu} >$$

and show that it's orientated parallel to $\vec{\mathrm{H}}$

c) Obtain from the calculations in b)

$$\mathcal{X} = \frac{1}{N}\frac{\partial M}{\partial H}$$

with $M = \|\vec{M}\|$ and $H = \|\vec{H}\|$

d) Show that the model describes a paramagnet e.g. that $\mathcal{X} > 0$ and find the behavior of $\mathcal{X}$ at high temperatures

2.2 Quantum

Dieteric equation of state(6pts)
The equation is given by

$$P(v\text{-}Nb) = Nk_B T e^{\frac{-aN}{Vk_B T}}$$

Where P, V, T are pressure, volume and temperature. a and b are some positive real numbers. Find P_c, T_c and V_c for this equation of state.

N particles(7pts)
Consider N distuinguisable and non-interacting particles. The single particle energy-spectrum is $\epsilon_n = n\epsilon$ where $\epsilon > 0$. The degenary of the n-th state is $g_n = n + 1$. Compute the partition function for the N-particle system and <E>. Evaluate the average E-fluctations defined by $< (\Delta E)^2 >=< E^2 > - < E >^2$

Quantum gasses(7pts)
Consider a 3D quantum gas of bosons or fermions. Assume that teh single particle energy is given by $\epsilon = p^\alpha$ where p is the absolute value of the momentum of the particle and $\alpha > 0$ and real. Using the grand canonical partitionfunction, show the following relation:

$$PV = \frac{\alpha}{3}E$$